surface and would not indicate the presence or absence of surface layers. Therefore, it is not appropriate to classify the surface material on the basis of these effective dielectric constants.

Pettengill and Henry[7] assume the conductivity s to be zero and the permeability u to be unity and show that $e = 2.81$ for $(R/D_s^2) = 0.064$. Radio temperature measurements by Salomonovich[8] lead him to deduce the maximum value for e as 1.5. Senior and Siegel[9] use the plane wave power reflection factor R_1^2 from a dielectric slab under normal incidence conditions

$$R^2 = \left[\frac{1 - [(u_0/u)e_e]^{1/2}}{1 + [(u_0/u)e_e]^{1/2}}\right]^2 \quad (3)$$

and use a surface model made up of corner reflectors, conelike projections, and flat portions to arrive at

$$e/u = 7.6 \times 10^{-6} \text{ mhos}^2$$

$$s/u = 2.7 \times 10^2 \text{ mhos/henry}$$

Numerous assumptions and "aesthetic" considerations used in their calculations reduce the value of their results to mere estimates although their approximated values, $e = 1.1\ e_0 = 9.6 \times 10^{-12}$ farads/m and $s = 3.4 \times 10^{-4}$ mhos/m for $u = u_0$, seem to fall in the range of other reported values.

The voltage reflection coefficient R for a sphere of radius a including an outer layer of thickness d is[10]

$$R = \frac{R_1 + R_2 \exp(-j2k_2d)}{1 + R_1R_2 \exp(-j2k_2d)} \quad (4)$$

where

R_1 = voltage reflection coefficient of the outer layer
R_2 = voltage reflection coefficient of the inner core
k_2 = $2\pi/L_2$, wave number
d = thickness of the outer layer on the sphere

and R_1 and R_2 are defined [from Eq. (3)] in terms of the dielectric constants as

$$R_1 = (1 - a)/(1 + a) \quad (5)$$

$$R_2 = (a - b)/(a + b) \quad (6)$$

where

$$a^2 = (u_0e_1/u_2e_0)[1 + j(s_1/w\ e_1)]$$

$$b^2 = (u_0e_2/u_2e_0)[1 + (s_1/w\ e_2)]$$

and where w is frequency in radians per second. These expressions can be used to solve for the six dielectric constants and the outer layer depth d from the previously suggested multifrequency experiment performed using a lunar orbiter type of satellite.

The depolarization of the electromagnetic waves incident at various angles on rough surfaces gives some indication of the type of roughness. In particular, linearly polarized incident signals become elliptically polarized by reflection from an absorbing medium, and this property may be used to determine the absorption coefficient of the lunar surface. Then the dielectric properties, the depth of the top surface layer, and the absorption coefficient could be correlated to determine the porosity and other physical properties of the surface material.

Pettengill and Henry[7] postulate that a relative permittivity of 2.81, calculated for their radar data taken at 68 cm wavelength, is similar to that of dry sand. Later experimental results (Evans and Pettengill[2]) seem to support this. Small-scale roughness indicated by $s/L = 0.1$ and $B/L = 1$ obtained (Hayre[3-5]) from Pettengill's results also seems to suggest that the lunar surface may have deformities that on the average may be approximately 68 cm in length and up to about 20 cm in height. It must be concluded that there is no unique method of definitely determining whether there is a dust layer or a sandy surface layer on the moon from either monostatic or bistatic radar studies or from albedo, temperature, and photometric function measurements. Nevertheless, such results as approximate depth of the top layer and the dielectric constants of the surface material may yield sufficient information to verify the design criterion for the Surveyor landing.

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Aerodynamic Coefficients in the Slip and Transition Regime

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Nomenclature

C_A = axial force coefficient
C_D = drag coefficient
C_M = pitching moment coefficient
D = base diameter
d = nose diameter
K_n = Knudsen number
M = Mach number, freestream
P = probability of no intermolecular collisions
Re = Reynolds number, freestream
U = velocity, freestream
X = aerodynamic coefficient
α = angle of attack
β = Martino number
δ^* = boundary-layer displacement thickness
Δ = shock detachment thickness
λ = mean free path
ρ = density

Subscripts

con = continuum
fm = free molecule
s = stagnation value
1 = freestream value

THE calculation of aerodynamic coefficients in the slip and transition regime is largely an inexact science. Some

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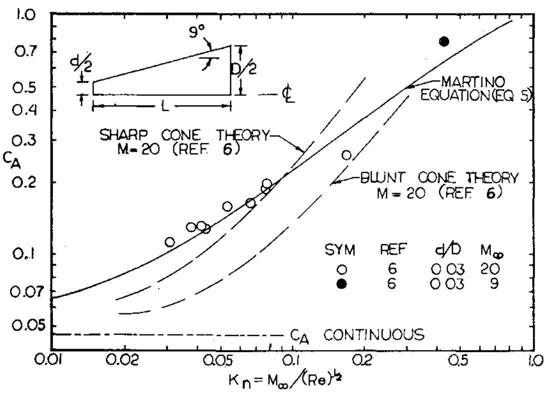

Fig. 1 Zero-lift drag of flat-nosed 9° cone.

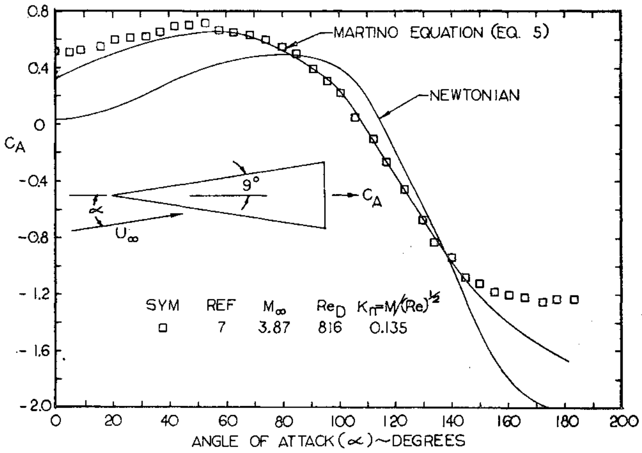

Fig. 2 Axial force coefficient for 9° cone.

theoretical solutions have been found, but these were generally restricted to either end of the slip or transition regime. The complexity of the problem has resulted in much emphasis placed on experimental results and empirical correlations.

A semiempirical correlation first proposed by Martino[1] in a study of temperature-recovery factors on circular cylinders and later used by Reeves and Van Camp[2] to predict heating effects seems to give promise of being quite useful in predicting aerodynamic coefficients of all types in the slip and transition regime.

To apply Martino's method to an estimation of aerodynamic coefficients, the coefficient can be thought of as being made up of two parts. Those molecules that pass through the gas layer adjacent to the body without a collision are assumed to exert an influence on the body proportional to the free molecule aerodynamic coefficient. On the other hand, those molecules that collide with the molecules in the gas layer are assumed to exert an influence on the body proportional to the continuum aerodynamic coefficient. The resultant expression for the aerodynamic coefficient is then

$$X = PX_{fm} + (1 - P)X_{con} \tag{1}$$

where P represents the fraction of molecules that pass through the gas layer without colliding with another molecule before striking the body surface, or, stated another way, P is the probability that a given molecule will collide with the adjacent surface before colliding with another molecule.

The boundary conditions on the probability P are

$$\lim P = 0 \qquad K_n \to 0$$

$$\lim P = 1 \qquad K_n \to \infty$$

The simplest mathematical expression that will satisfy the boundary conditions on P is

$$P = (K_n)/(1 + K_n) \tag{2}$$

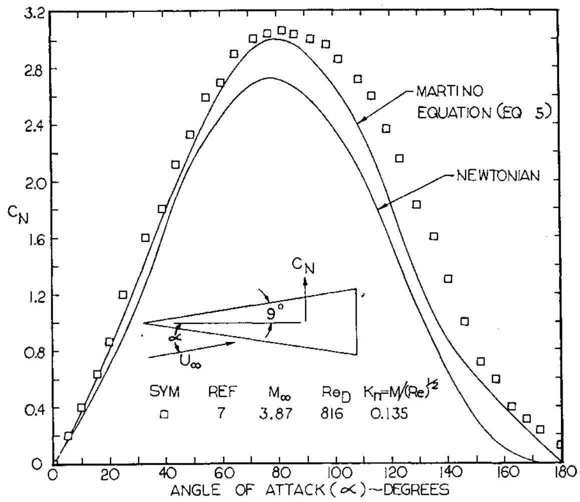

Fig. 3 Normal force coefficient for 9° cone.

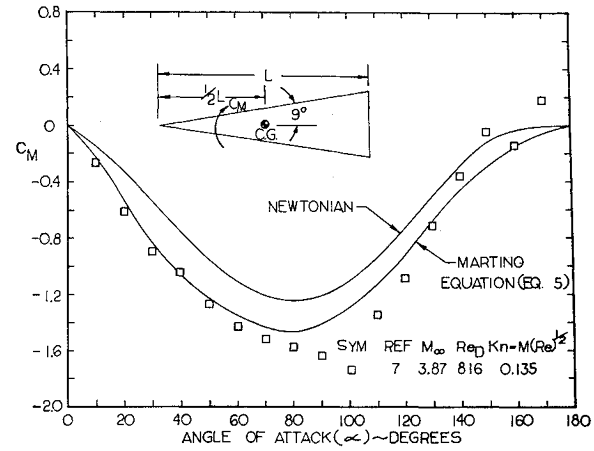

Fig. 4 Pitching moment coefficient for 9° cone.

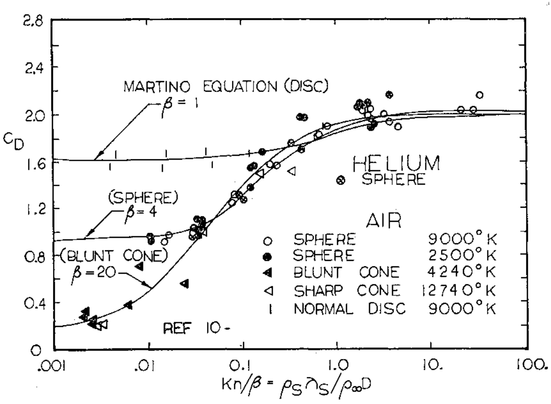

Fig. 5 Drag coefficients of various shapes.

Substituting Eq. (2) into Eq. (1),

$$X = (X_{con} + K_n X_{fm})/(1 + K_n) \tag{3}$$

Equation (3) is Martino's semiempirical correlation equation applied to aerodynamic coefficients. The success of this equation seems to hinge strongly on how one defines the Knudsen number.

In the slip regime ($0.01 < K_n$ 0.1), experience has shown that the Knudsen number should be based on the thickness of the boundary-layer displacement thickness (see, e.g., Talbot[3] and Tsien[4]). The well-known expression for the Knudsen number in this regime is

$$K_n = \lambda_1/\delta^* \simeq M/Re^{1/2} \tag{4}$$

Substituting Eq. (4) into Eq. (3), one obtains Martino's equation for the slip regime:

$$X = [X_{\text{con}} + (M/Re^{1/2})(X_{fm})]/[1 + (M/Re^{1/2})] \quad (5)$$

Figures 1–4 show how Martino's equation [Eq. (5)] compares with experimental data. The free molecule aerodynamic coefficients used in Eq. (5) were obtained from Blick.[5] Figure 1 shows a comparison between Martino's equation and the method of Lukasiewicz et al.,[6] which is essentially a combination of Bertram's[8] viscous interaction skin friction correction and Li and Nagamatsu's[9] induced pressure correction modified by the Mangler transformation.

In the transition regime, the Knudsen number given by Eq. (4) is probably not the best one to use. If one assumes that the characteristic length is proportional to the shock-detachment thickness and the mean free path evaluated behind the shock, then the Knudsen number can be defined as

$$K_n = \beta\lambda_s/\Delta = \beta\rho_s\lambda_s/\rho_1 D \quad (6)$$

If the stagnation temperature is low, then $\lambda\rho$ is insensitive to temperature, and Eq. (6) would reduce to

$$K_n = \beta\lambda_1/D \quad (7)$$

β, defined here to be the "Martino number," is simply a numerical factor that fits the Martino equation [Eq. (3)] as close as possible to the experimental data. Experimental drag data from Bloxsom and Rhodes[10] were correlated by Martino's equation [Eq. (3)], along with Eq. (6), in Fig. 5. Each shape in Fig. 5 had a different Martino number. It was found that the Martino numbers could be correlated by the following equation:

$$\beta = \exp[3.36 - 4.26\,(C_{D_{\text{con}}})/(C_{D fm})] \quad (8)$$

It is not known at this time whether the drag Martino number given by Eq. (8) is applicable to other aerodynamic coefficients. If it is, then one can simply substitute the continuum to free molecule ratio of the coefficient into Eq. (8) in place of the drag coefficient ratio. However, further experimental data on coefficients (other than drag coefficients) will have to be obtained before the validity of his method can be checked.

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Flapping Propulsion Wake Analysis

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A WING, oscillating normal to its direction of flight, experiences a propulsive force that can be evaluated from its wake characteristics. A classic wake formula is adapted to relate thrust force to the flapping frequency and forward speed, disclosing important distinctive features of unsteady propulsion. It is shown that a portion of the thrust remains finite as forward speed tends to zero. Aside from its interest for natural and low-speed flight, the result is applicable in such domains as underwater propulsion, by reason of the fact that the thrust mechanism is independent of normal (i.e., lift) force.

Unsteady wing propulsion, once termed Katzmayr effect, furnishes a specific physical representation as well as a useful terminology; the propulsor will be henceforth referred to as a wing, although the discussion applies equally well to various vortex-shedding configurations. A variety of wing and flap oscillation modes are known which approach ideal mechanical efficiency, and, in all cases, a thick wake is formed which consists of the flow region bounded by two staggered rows of oppositely directed vorticity. Except for the fact that the vortex sense is reversed (hence, also the direction of the proper vortex motion), the vortex pattern is identical to the vortex street of Bénard and Kármán. The thrust force is thus given directly by a slight modification of Kármán's formula:

$$T = \rho V\Gamma \frac{h}{l} + \rho \frac{\Gamma^2}{l}\left(\frac{h/l}{2^{1/2}} - \frac{1}{2\pi}\right) \quad (1)$$

where Γ is the vortex strength, V is the forward speed, and h/l is the ratio of street width to streamwise vortex spacing in either row (see, e.g., Ref. 1). The latter ratio being a known constant, it is convenient to regard the parameter l as a measure of wing oscillation amplitude.

The circulation Γ can be replaced by the cyclic frequency f of wing oscillation by noting that this is the ratio of speed $V + u_T$ of vortex relative to wing divided by spacing l:

$$f = (V + u_T)/l$$

The motion u_T of the vortex relative to the freestream is known from vortex dynamics as

$$u_T = \frac{1}{2(2)^{1/2}} \frac{\Gamma}{l}$$

so that

$$f = \frac{V}{l}\left\{1 + \frac{1}{2(2)^{1/2}} \frac{\Gamma}{lV}\right\} \quad (2)$$

Substitution for Γ in (1) gives the thrust dependence on frequency and forward speed:

$$T = \rho V \frac{h}{l}\,(2)^{1/2} l(fl - V) + \rho l \cdot 8 \left(\frac{h/l}{2^{1/2}} - \frac{1}{2\pi}\right)(fl - V)^2 \quad (1')$$

Although both terms on the right side of (1′) include the familiar velocity-squared terms, the second term is also seen to contain a contribution to the thrust, which is independent of forward speed. This feature is the principal reason for the importance of oscillating wings in low-speed flight technology. Thrust being independent of lift, moreover, it ap-

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